Merging Incommensurable Possibilistic *DL-Lite* Assertional Bases

S. Benferhat[1] Z. Bouraoui[1] S. Lagrue[1] J. Rossit[2]

[1] CRIL-CNRS, Univ. d'Artois, {benferhat,bouraoui,lagrue}@cril.fr,

[2] LIPADE, Univ Paris Descartes, julien.rossit@parisdescartes.fr

Motivations

3 main notions

- Merging multiple-source uncertain information
- Incommensurability of uncertainty scales
  - Assessment marks
    - marked on the 0-100 scale
    - marked on the 0-20 scale
    - Using qualitative scale : A+, A, A-, etc
- Lightweight ontologies (DL-lite)

Why lightweight DL?

Which language to use?

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Nice features of DL-Lite

- A reasonable expressive language
- DL-lite logics are appropriate for applications where queries need to be efficiently handled
- Tractable methods for computing conflicts.

DL-lite

DL-lite: vocabulary

The starting points are N_C, N_R and N_I, three pairwise disjoint sets :

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Let a and b be two individuals. An ABox is a set of:

- Membership assertions on atomic concepts:

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DL-Lite$_{core}$ TBox consists of a set of concept inclusion assertions:

$$B_1 \sqsubseteq B_2, \qquad B_1 \sqsubseteq \neg B_2,$$

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$$B_i \longrightarrow \quad A \mid \exists P \mid \exists P^{-}$$

Problem : merging DL-Lite$^{\pi}$

Contexte

- DL-Lite$^{\pi}$: $\mathcal{K} = \mathcal{T} \cup \mathcal{A} = \{(\phi, \alpha) : \phi \in$ DL-Lite and $\alpha \in]0, 1]\}$

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- Each $\mathcal{T}_i \cup \mathcal{A}_i$ is consistent
- Sources do not share the same uncertainty scale

Possibilistic fusion with commensurability

Principle

- If $\mathcal{A}_1 \cup \mathcal{A}_2 \cup \ldots \cup \mathcal{A}_n$ is consistent with $\mathcal{T}$, then

$$\Delta_{\pi}^{\mathcal{T}}(E) = \mathcal{T} \cup \mathcal{A}_1 \cup \mathcal{A}_2 \cup \ldots \cup \mathcal{A}_n$$

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- Combine $\pi_i^{'}$s (with the minimum operation) to select the result of merging.

Possibilistic merging

Example

- $\mathcal{T}$=$\{A \sqsubseteq B,\ B \sqsubseteq \neg C\}$
- $\mathcal{A}_1$=$\{(A(a), .6)\ (C(b), .5)\}$
- $\mathcal{A}_2$=$\{(C(a), .4)\ (B(b), .8), (A(b), .7)\}$.

$\mathcal{I}$	$\cdot^{\mathcal{I}}$	$\pi_{\mathcal{A}_1}$	$\pi_{\mathcal{A}_2}$	$\Delta^{min}_{\mathcal{T}}(\mathcal{A})$
$\mathcal{I}_1$	A={a},B={a},C={b}	1	.2	.2
$\mathcal{I}_2$	A={},B={},C={a,b}	.4	.2	.4
$\mathcal{I}_3$	A={a,b},B={a,b},C={}	.5	.6	**.5**
$\mathcal{I}_4$	A={b},B={b},C={a}	.4	1	.4

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- $[\Delta_{\mathcal{T}}^{min}(\mathcal{A})] = \mathcal{I}_3$

At the syntactic level

Method

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- Computing $\Delta_{\pi}^{\mathcal{T}}(E)$ is done in a polynomial time.
- Question:
  How to extend the possibilistic merging when the uncertainty scales are incommensurable?

Compatible-based merging

Principle

Incommensurable merging

$=$

Family of compatible and commensurable merging

Example

$$\begin{array}{lll} \mathcal{T} & = & \{A \sqsubseteq B,\ B \sqsubseteq \neg C\} \\ \mathcal{A}_1 & = & \{(A(a), .6)\ (C(b), .5)\} \\ \mathcal{A}_2 & = & \{(C(a), .4)\ (B(b), .8),\ (A(b), .7)\}. \end{array}$$

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Semantic fusion

- Define a partial pre-order over interprétations

$$\mathcal{I} <_{\forall}^{\mathcal{A}} \mathcal{I}' \iff \forall \mathcal{R} \in \mathcal{R}(\mathcal{A}),\ \mathcal{I} \triangleleft_{Min}^{\mathcal{A}^{\mathcal{R}}} \mathcal{I}'$$

- Select the best ones to define the result of merging

$$Mod(\Delta_{\mathcal{T}}^{\forall}(\mathcal{A}))=\{\mathcal{I} \in Mod(\mathcal{T}) \colon \nexists\, \mathcal{I}' \in Mod(\mathcal{T}), \mathcal{I}' <_{\forall}^{\mathcal{A}} \mathcal{I}\}$$

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Exemple

Example (continued)

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- $\mathcal{A}_2^{\mathcal{R}_1} = \{(C(a), .2), (B(b), .9), (A(b), .6)\}$

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$\mathcal{I}$	$\nu_{\mathcal{A}^{\mathcal{R}_1}}(\mathcal{I})$	Min	$\nu_{\mathcal{A}^{\mathcal{R}_2}}(\mathcal{I})$	Min
$\mathcal{I}_1$	$< 1, .1 >$	.1	$< 1, .4 >$	.4
$\mathcal{I}_2$	$< .2, .1 >$	.1	$< .6, .4 >$	.4
$\mathcal{I}_3$	$< .6, .8 >$	**.6**	$< .8, .7 >$	**.7**
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$Mod(\Delta^{\forall}_{\mathcal{T}}(\mathcal{A}))$ can be directly computed from A_i's in a polynomial time. Thanks to the facts:

- A conflict necessarily implies:
  - One NI axiom.
  - One or two membership assertions.
- A polynomial time algorithm to compute conflicts

Selecting one compatible scale

Normalisation

- $\alpha_{\mathcal{A}_i}$: set of degrees in $\mathcal{A}_i$
- $min_{\mathcal{A}_i}$ (resp. $max_{\mathcal{A}_i}$) is the minimum (maximum) degree used in $\alpha_{\mathcal{A}_i}$

$$N(\alpha_i) = \frac{\alpha_i - (min_{\mathcal{A}_i} - \epsilon)}{max_{\mathcal{A}_i} - (min_{\mathcal{A}_i} - \epsilon)}$$

- $\alpha_i \in \alpha_{\mathcal{A}_i}$ and $0 < \epsilon < min_{\mathcal{A}_i}$

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- $\mathcal{A}_1$={$(A(a), 1)$, $(C(b), .09)$}
- $\mathcal{A}_2$={$(C(a), 0, 02)$, $(B(b), 1)$, $(A(b), .75)$}

Conclusions

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(CSS) $\forall B_i \in E$, if $B_i^* \models \mu$, then $B_i^* \wedge \triangle^{\mu}(E)$ inconsistant
  - In the propositional setting, no way to satisfy CCS when only selecting one compatible scale.

Conclusions

- Safe possibilistic DL-Lite KB Merging without commensurability assumption using compatible scales

- Merging in DL-lite$^{\pi}$ setting is tractable while it is a hard in a (weighted) propositional setting

- Rational postulates for merging in DL-lite$^{\pi}$ setting

(CSS) $\forall B_i \in E$, if $B_i^* \models \mu$, then $B_i^* \wedge \triangle^{\mu}(E)$ inconsistant
  - In the propositional setting, no way to satisfy CCS when only selecting one compatible scale.
  - Is-it the case for DL-lite$^{\pi}$ setting.