Merging Incommensurable Possibilistic DL-Lite Assertional Bases

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Motivations

3 main notions

- Merging multiple-source uncertain information

- Incommensurability of uncertainty scales
  - Assessment marks
    - marked on the 0-100 scale
    - marked on the 0-20 scale
    - Using qualitative scale : A+, A, A-, etc

- Lightweight ontologies (DL-lite)
Why lightweight DL?

Which language to use?

- Each knowledge base format is suitable for some applications
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### Why lightweight DL?

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- In general, the more expressive is the language the more hard is its inference relations.
- Always, one needs to reach for a good compromise between expressiveness and computational issues.
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Nice features of DL-Lite

- A reasonable expressive language
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### Why lightweight DL?

#### Which language to use?
- Each knowledge base format is suitable for some applications.
- In general, the more expressive is the language the more hard is its inference relations.
- Always, one needs to reach for a good compromise between expressiveness and computational issues.

#### Nice features of DL-Lite
- A reasonable expressive language
- DL-lite logics are appropriate for applications where queries need to be efficiently handled
- Tractable methods for computing conflicts.
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Let $a$ and $b$ be two individuals. An ABox is a set of:
- Membership assertions on atomic concepts:
  \[ A(a) \]
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- Membership assertions on atomic concepts:
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- membership assertions on atomic roles:
  \[ P(a, b) \]
To define complex concepts and roles:

- ¬ (negated concepts or roles)
- ∃ (set of individuals obtained by projection on the first dimension of a role)
- (inverse relation)
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### DL-lite: unary connectors

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### TBOX of DL-lite\(_{\text{core}}\)

DL-Lite\(_{\text{core}}\) TBox consists of a set of concept inclusion assertions:

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B_1 \sqsubseteq B_2, \quad B_1 \sqsubseteq \neg B_2,
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B_i \rightarrow A \mid \exists P \mid \exists P
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Problem: merging DL-Lite$^\pi$

Contexte

- **DL-Lite$^\pi$:** $\mathcal{K} = \mathcal{T} \cup \mathcal{A} = \{ (\phi, \alpha) : \phi \in \text{DL-Lite} \text{ and } \alpha \in ]0, 1] \}$

Assumptions

- Sources share the same ontology: $\mathcal{T}_1 = \ldots = \mathcal{T}_n$
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### Principle

- If $\mathcal{A}_1 \cup \mathcal{A}_2 \cup \ldots \cup \mathcal{A}_n$ is consistent with $\mathcal{T}$, then

$$\Delta_{\pi}^{\mathcal{T}}(E) = \mathcal{T} \cup \mathcal{A}_1 \cup \mathcal{A}_2 \cup \ldots \cup \mathcal{A}_n$$
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- For each source $i$, rank-order the interpretations $\mathcal{I}$ with respect to the highest assertion that is rejected from $A_i$. 
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- More precisely:
  
  $$\pi_i(\mathcal{I}) = 1 - \max\{f : f \in \mathcal{A}_i, \mathcal{I} \not\models f\}.$$
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- More precisely:
  
  \[
  \pi_i(\mathcal{I}) = 1 - \max\{f : f \in \mathcal{A}_i, \mathcal{I} \nmid f\}.
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- Combine $\pi_i$'s (with the minimum operation) to select the result of merging.
Possibilistic merging

Example

- $\mathcal{T} = \{ A \subseteq B, B \subseteq \neg C \}$
- $\mathcal{A}_1 = \{ (A(a), .6) \ (C(b), .5) \}$
- $\mathcal{A}_2 = \{ (C(a), .4) \ (B(b), .8), \ (A(b), .7) \}$.

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<th>$\pi_{\mathcal{A}_1}$</th>
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<th>$\Delta_{\mathcal{T}}^{\text{min}}(\mathcal{I})$</th>
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<td>$\mathcal{I}_1$</td>
<td>$A={a}, B={a}, C={b}$</td>
<td>1</td>
<td>.2</td>
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</tr>
<tr>
<td>$\mathcal{I}_2$</td>
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</tr>
<tr>
<td>$\mathcal{I}_3$</td>
<td>$A={a,b}, B={a,b}, C={}$</td>
<td>.5</td>
<td>.6</td>
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</tr>
<tr>
<td>$\mathcal{I}_4$</td>
<td>$A={b}, B={b}, C={a}$</td>
<td>.4</td>
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- $[\Delta^\text{min}_\mathcal{T}(\mathcal{A})] = \mathcal{I}_3$
At the syntactic level

Method

1. Define: \( \mathcal{A}_\oplus = \mathcal{A}_1 \cup \mathcal{A}_2 \cup \ldots \cup \mathcal{A}_n \)
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3. \( \Delta_T^\pi(E) = T \cup \{ (\phi, \alpha) : (\phi, \alpha) \in A_\oplus \text{ and } \alpha > x \} \)

Remarks

- Computing \( \Delta_T^\pi(E) \) is done in a polynomial time.
- Question: How to extend the possibilistic merging when the uncertainty scales are incommensurable?
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Compatible-based merging

**Principle**

Incommensurable merging

= 

Family of compatible and commensurable merging

**Example**

\[ \mathcal{T} = \{ A \sqsubseteq B, B \sqsubseteq \neg C \} \]

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Semantic fusion

- Define a partial pre-order over interprétations

\[ \mathcal{I} \prec^\mathcal{A} \mathcal{I}' \iff \forall \mathcal{R} \in \mathcal{R}(\mathcal{A}), \mathcal{I} \prec^\mathcal{R} \text{Min} \mathcal{I}' \]

- Select the best ones to define the result of merging

\[ \text{Mod}(\Delta^\forall_T(\mathcal{A})) = \{ \mathcal{I} \in \text{Mod}(T) : \exists \mathcal{I}' \in \text{Mod}(T), \mathcal{I}' \prec^\mathcal{A} \mathcal{I} \} \]
Semantic fusion

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\[ \text{Mod}(\Delta_T^\forall(A)) = \{ I \in \text{Mod}(T): \not\exists I' \in \text{Mod}(T), I' <^\forall I \} \]
Example (continued)

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• $A_1^{R_1} = \{(A(a), .8), (C(b), .4)\}$
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<th>( \nu_{A_{R1}^1}(I) )</th>
<th>Min</th>
<th>( \nu_{A_{R2}^1}(I) )</th>
<th>Min</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I_1 )</td>
<td>&lt; 1, .1 &gt;</td>
<td>.1</td>
<td>&lt; 1, .4 &gt;</td>
<td>.4</td>
</tr>
<tr>
<td>( I_2 )</td>
<td>&lt; .2, .1 &gt;</td>
<td>.1</td>
<td>&lt; .6, .4 &gt;</td>
<td>.4</td>
</tr>
<tr>
<td>( I_3 )</td>
<td>&lt; .6, .8 &gt;</td>
<td>.6</td>
<td>&lt; .8, .7 &gt;</td>
<td>.7</td>
</tr>
<tr>
<td>( I_4 )</td>
<td>&lt; .2, 1 &gt;</td>
<td>.2</td>
<td>&lt; .6, 1 &gt;</td>
<td>.6</td>
</tr>
</tbody>
</table>
Example (continued)

- \( \mathcal{A}_1 = \{(A(a), .6), (C(b), .5)\} \)
- \( \mathcal{A}_2 = \{(C(a), .4), (B(b), .8), (A(b), .7)\} \)
- \( \mathcal{A}_{R1}^1 = \{(A(a), .8), (C(b), .4)\} \)
- \( \mathcal{A}_{R1}^2 = \{(C(a), .2), (B(b), .9), (A(b), .6)\} \)
- \( \mathcal{A}_{R2}^1 = \{(A(a), .4), (C(b), .2)\} \)
- \( \mathcal{A}_{R2}^2 = \{(C(a), .3), (B(b), .6), (A(b), .5)\} \)

<table>
<thead>
<tr>
<th>( \mathcal{I} )</th>
<th>( \nu_{\mathcal{A}_{R1}}(\mathcal{I}) )</th>
<th>Min</th>
<th>( \nu_{\mathcal{A}_{R2}}(\mathcal{I}) )</th>
<th>Min</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathcal{I}_1 )</td>
<td>(&lt; 1, .1 &gt;)</td>
<td>.1</td>
<td>(&lt; 1, .4 &gt;)</td>
<td>.4</td>
</tr>
<tr>
<td>( \mathcal{I}_2 )</td>
<td>(&lt; .2, .1 &gt;)</td>
<td>.1</td>
<td>(&lt; .6, .4 &gt;)</td>
<td>.4</td>
</tr>
<tr>
<td>( \mathcal{I}_3 )</td>
<td>(&lt; .6, .8 &gt;)</td>
<td>.6</td>
<td>(&lt; .8, .7 &gt;)</td>
<td>.7</td>
</tr>
<tr>
<td>( \mathcal{I}_4 )</td>
<td>(&lt; .2, 1 &gt;)</td>
<td>.2</td>
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</tr>
</tbody>
</table>
Important computational result

$\text{Mod}(\Delta^\forall_T(A))$ can be directly computed from $A_i$'s in a polynomial time.
Important computational result

$\text{Mod}(\Delta_F^\forall(A))$ can be directly computed from $A_i$’s in a polynomial time. Thanks to the facts:

- A conflict necessarily implies:
  - One NI axiom.
  - One or two membership assertions.
- A polynomial time algorithm to compute conflicts
Selecting one compatible scale

Normalisation

- $\alpha_{A_i}$: set of degrees in $A_i$
- $\min_{A_i}$ (resp. $\max_{A_i}$) is the minimum (maximum) degree used in $\alpha_{A_i}$

$$N(\alpha_i) = \frac{\alpha_i - (\min_{A_i} - \epsilon)}{\max_{A_i} - (\min_{A_i} - \epsilon)}$$

- $\alpha_i \in \alpha_{A_i}$ and $0 < \epsilon < \min_{A_i}$
Normalisation

Example

• $A_1=\{(A(a), .6), (C(b), .5)\}$
• $A_2=\{(C(a), .4), (B(b), .8), (A(b), .7)\}$
Normalisation

Example

- $A_1=\{(A(a), .6), (C(b), .5)\}$
- $A_2=\{(C(a), .4), (B(b), .8), (A(b), .7)\}$

$$\min_{A_1} = .5, \min_{A_2} = .4, \max_{A_1} = .6, \max_{A_2} = .8 \text{ et } \epsilon = .01$$
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Example

- $A_1 = \{(A(a), .6), (C(b), .5)\}$
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$$\min_{A_1} = .5, \min_{A_2} = .4, \max_{A_1} = .6, \max_{A_2} = .8 \text{ et } \epsilon = .01$$

- $A_1 = \{(A(a), 1), (C(b), .09)\}$
- $A_2 = \{(C(a), 0.02), (B(b), 1), (A(b), .75)\}$
• Safe possibilistic DL-Lite KB Merging without commensurability assumption using compatible scales
Conclusions

- Safe possibilistic DL-Lite KB Merging without commensurability assumption using compatible scales

- Merging in DL-lite$^\pi$ setting is tractable while it is a hard in a (weighted) propositional setting

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  - Is-it the case for DL-lite$^\pi$ setting.