From Classical to Consistent Query Answering under Existential Rules

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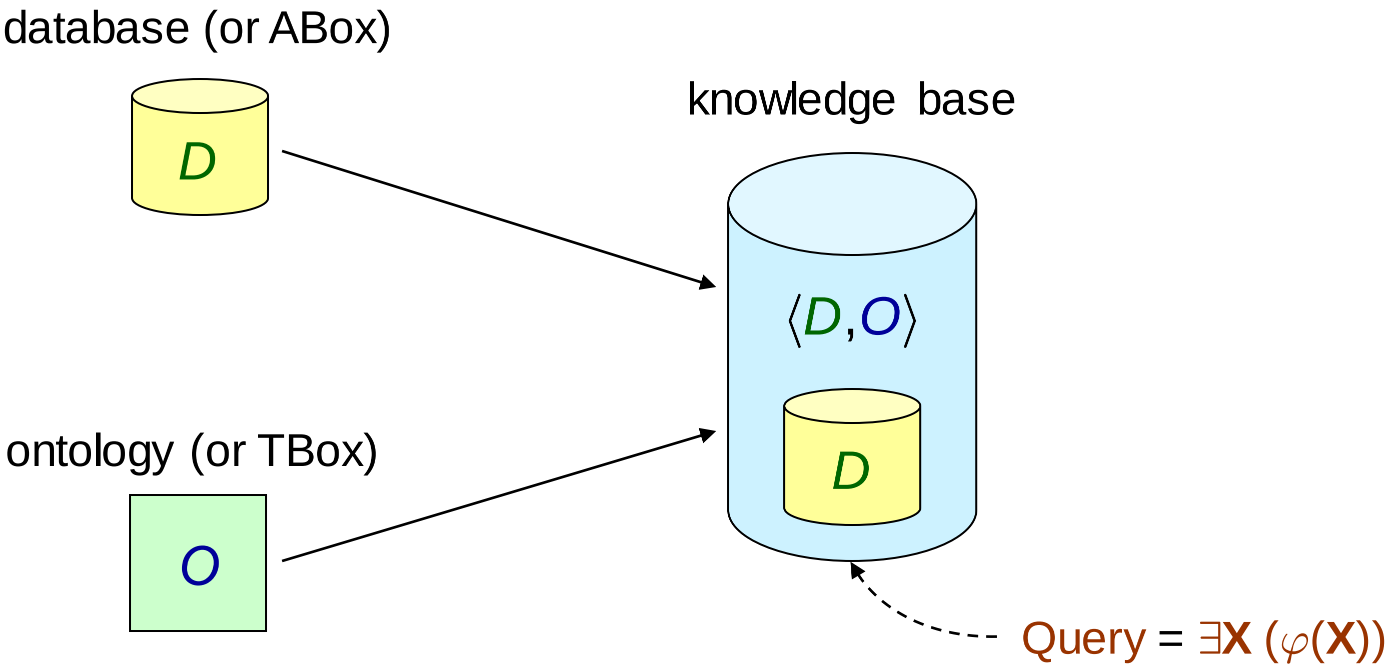
Ontology-based Query Answering (OBQA)



$$\langle D, O \rangle \models \text{Query} \quad \Leftrightarrow \quad D \wedge O \models \text{Query}$$

A Simple Example

D =

professor(John)

fellow(John)

O =

$\forall X\ (\mathit{professor}(X) \rightarrow \exists Y\ (\mathit{faculty}(X) \wedge \mathit{teaches}(X,Y)))$

$\forall X\ (\mathit{fellow}(X) \rightarrow \mathit{faculty}(X))$

$\forall M \models \langle D, O \rangle\ :\ M$ = **...** *teaches*(John,#) **...**

{John $\rightarrow$ John, $X \rightarrow$ #}

$\exists X\ (\mathit{teaches}(\text{John}, X))$ ✓

A Simple Example

D =

professor(John)

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O =

$\forall X\ (professor(X) \rightarrow \exists Y\ (faculty(X) \wedge teaches(X,Y)))$

$\forall X\ (fellow(X) \rightarrow faculty(X))$

$\forall X\ (professor(X) \wedge fellow(X) \rightarrow \bot)$

no model $\Rightarrow$ every query is entailed

Handling Data Inconsistencies

- The data are likely to be inconsistent with the ontology

- Standard semantics fails: everything is inferred - not meaningful answers

- Two approaches to inconsistency-handling:
  - Resolve the inconsistencies - ideal, but not always possible
  - Live with the inconsistencies - inconsistency-tolerant semantics

ABox Repair (AR) Semantics

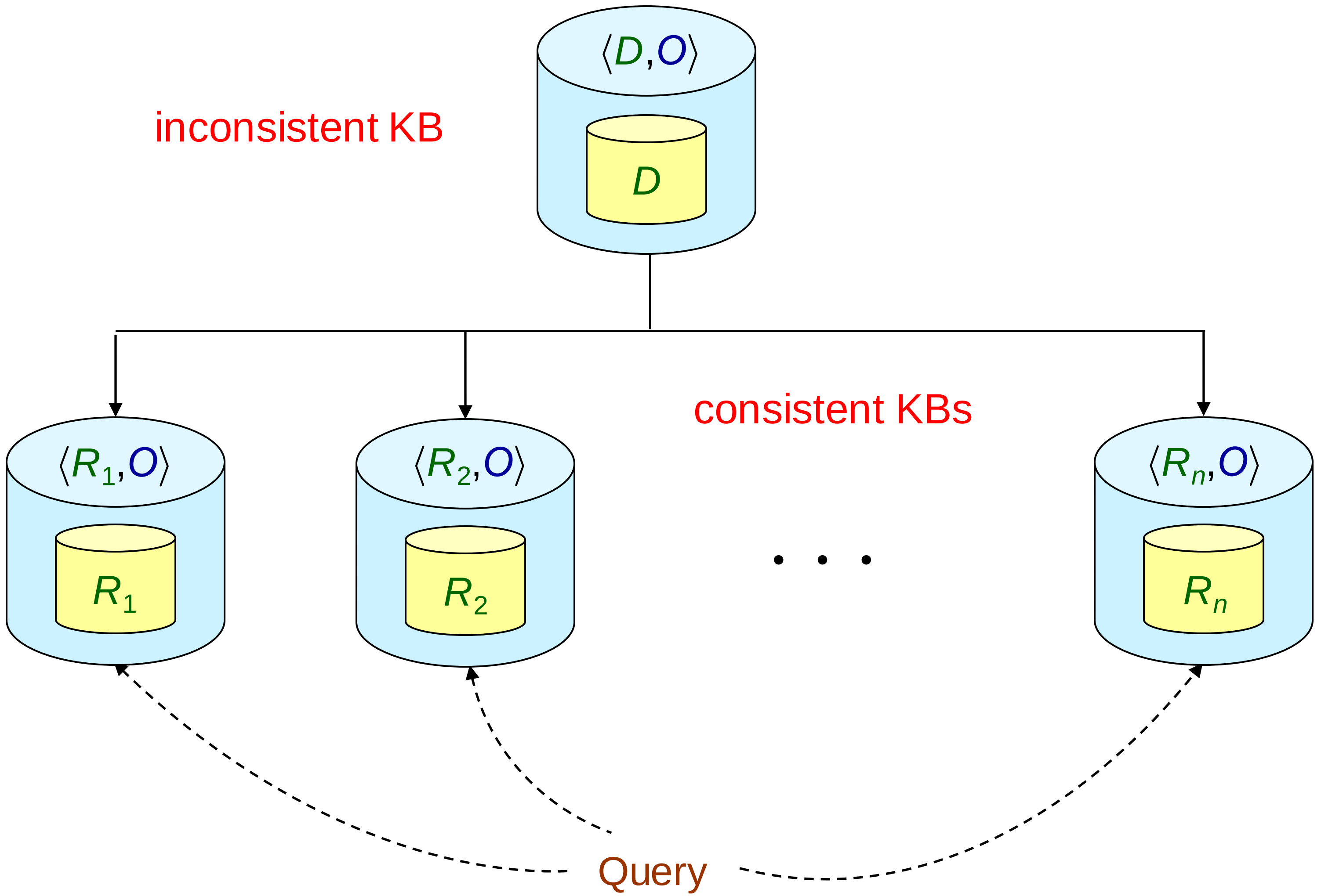
- Standard inconsistency-tolerant semantics

- **IDEA:** The query must be entailed by every database repair

$\subseteq$-maximal consistent subsets of the database

[Lembo et al., RR 2010]

ABox Repair (AR) Semantics



$$\langle D,O \rangle \models_{AR} \text{Query} \quad \Leftrightarrow \quad \forall R \in \{R_1,\ldots,R_n\}: \langle R,O \rangle \models \text{Query}$$

ABox Repair (AR) Semantics: Example

D =

professor(John)

fellow(John)

O =

$\forall X\, (professor(X) \rightarrow \exists Y\, (faculty(X) \wedge teaches(X,Y)))$

$\forall X\, (fellow(X) \rightarrow faculty(X))$

$\forall X\, (professor(X) \wedge fellow(X) \rightarrow \bot)$

$\langle D, O \rangle \models_{\mathrm{AR}} faculty(\mathrm{John})$ ✓

R_1 = *professor*(John)

$\langle R_1, O \rangle \models faculty(\mathrm{John})$ ✓

R_2 = *fellow*(John)

$\langle R_2, O \rangle \models faculty(\mathrm{John})$ ✓

ABox Repair (AR) Semantics: Example

D =

professor(John)

fellow(John)

O =

$\forall X\ (\mathit{professor}(X) \rightarrow \exists Y\ (\mathit{faculty}(X) \wedge \mathit{teaches}(X,Y)))$

$\forall X\ (\mathit{fellow}(X) \rightarrow \mathit{faculty}(X))$

$\forall X\ (\mathit{professor}(X) \wedge \mathit{fellow}(X) \rightarrow \bot)$

$\langle D,O \rangle \vDash_{\mathrm{AR}} \exists X\ (\mathit{teaches}(\mathrm{John},X))$ ✗

R_1 = *professor*(John)

$\langle R_1,O \rangle \vDash \exists X\ (\mathit{teaches}(\mathrm{John},X))$ ✓

R_2 = *fellow*(John)

$\langle R_2,O \rangle \vDash \exists X\ (\mathit{teaches}(\mathrm{John},X))$ ✗

AR Semantics

- Lots of recent work and complexity results for description logics

  [Lembo et al., RR 2010 / Rosati, IJCAI 2011 / Bienvenu, AAAI 2012 / Bienvenu & Rosati, IJCAI 2013]

- This talk is about existential rules + negative constraints

  [Lukasiewicz, Martinez & Simari, ODBASE 2013 / Lukasiewicz, Martinez, P. & Simari, AAAI 2015]

$$\forall \mathbf{X}\ (\varphi(\mathbf{X}) \rightarrow \exists \mathbf{Y}\ (\psi(\mathbf{X},\mathbf{Y}))) \quad + \quad \forall \mathbf{X}\ (\varphi(\mathbf{X}) \rightarrow \bot)$$

Our Goal

Perform an in-depth $\boxed{\text{complexity}}$ analysis of consistent query answering under the

main classes of existential rules + negative constraints

- Combined
- Bounded-arity combined
- Fixed-program combined
- Data

generic complexity results - from classical to consistent query answering

Combined Complexity

combined or ba-combined or fp-combined

class of $\exists$-rules

complexity class

$\mathbb{M}$ complexity of classical query answering under $\mathbb{L}$ is $\mathbb{C}$-complete

$$\Downarrow$$

$\mathbb{M}$ complexity of consistent query answering under $\mathbb{L}[\perp]$ is:

$\Pi_{P,2}$-complete if $\mathbb{C}$ = NP

$\mathbb{C}$-complete if $\mathbb{C} \supseteq$ PSPACE & $\mathbb{C}$ is deterministic

Combined Complexity: Upper Bounds

Guess and check algorithm (for the complement of the problem)

Input: D, $O \in \mathbb{L}[\perp]$, Q

1. Guess $R \subseteq D$ - a possible repair
2. Verify that R is a repair, i.e., $\langle R,O \rangle$ is consistent and R is $\subseteq$-maximal
3. Verify that $\langle R,O \rangle$ does not entail Q

no harder than classical query answering under $\mathbb{L}$

$$\Rightarrow \text{ our problem is in } \mathrm{coNP}^{\mathbb{C}} \Rightarrow \text{ in } \begin{cases} \mathrm{coNP}^{\mathrm{NP}} = \mathrm{co}\Sigma_{P,2} = \Pi_{P,2} & \text{if } \mathbb{C} = \mathrm{NP} \\ \mathrm{coNP}^{\mathbb{C}} = \mathrm{co}\mathbb{C} = \mathbb{C} & \text{if } \mathbb{C} \supseteq \mathrm{PSPACE},\ \mathbb{C} \text{ is deterministic} \end{cases}$$

Combined Complexity

combined or ba-combined or fp-combined   class of $\exists$-rules   complexity class

$\mathbb{M}$ complexity of classical query answering under $\mathbb{L}$ is $\mathbb{C}$-complete

$$\Downarrow$$

$\mathbb{M}$ complexity of consistent query answering under $\mathbb{L}[\perp]$ is:

$\Pi_{P,2}$-complete   if $\mathbb{C} = \mathrm{NP}$

$\mathbb{C}$-complete   if $\mathbb{C} \supseteq \mathrm{PSPACE}$ & $\mathbb{C}$ is deterministic

A Strong $\Pi_{P,2}$-hardness Result

Consistent query answering under the single constraint

$$\forall X \forall Y \forall Z \forall W \ (p(X,Y,Z) \wedge p(W,X,Z) \rightarrow \perp)$$

while the database and the query use only binary and ternary predicates

(by reduction from satisfiability of 2QBF formulas)

$$\Downarrow$$

For every class $\mathbb{L}$ of existential rules, the fp-combined complexity of consistent query answering under $\mathbb{L}[\perp]$ is $\Pi_{P,2}$-hard

Combined Complexity

combined or ba-combined or fp-combined

class of $\exists$-rules

complexity class

$\mathbb{M}$ complexity of classical query answering under $\mathbb{L}$ is $\mathbb{C}$-complete

$$\Downarrow$$

$\mathbb{M}$ complexity of consistent query answering under $\mathbb{L}[\perp]$ is:

$\Pi_{P,2}$-complete if $\mathbb{C} = \mathrm{NP}$

$\mathbb{C}$-complete if $\mathbb{C} \supseteq \mathrm{PSPACE}$ & $\mathbb{C}$ is deterministic

Data Complexity

class of ∃-rules

complexity class

data complexity of classical query answering under $\mathbb{L}$ is $\mathbb{C}$-complete

$$\Downarrow$$

data complexity of consistent query answering under $\mathbb{L}[\perp]$ is:

coNP-complete if $\mathbb{C} \subseteq$ PTIME

Data Complexity: Upper Bounds

Guess and check algorithm (for the complement of the problem)

Input: D, $O \in \mathbb{L}[\perp]$, Q

1. Guess $R \subseteq D$ - a possible repair

2. Verify that R is a repair, i.e., $\langle R, O \rangle$ is consistent and R is $\subseteq$-maximal

3. Verify that $\langle R, O \rangle$ does not entail Q

no harder than classical query answering under $\mathbb{L}$

$\Rightarrow$ our problem is in $\mathrm{coNP}^{\mathbb{C}}$ $\Rightarrow$ in coNP (since $\mathrm{NP}^{\mathrm{PTIME}} = \mathrm{NP}$)

A Strong coNP-hardness Result

Consistent query answering under the single constraint

$$\forall X \;\; (p(X) \wedge s(X) \; \rightarrow \; \bot)$$

while the query is fixed

(by reduction from 2+2UNSAT)

$$\Downarrow$$

For every class $\mathbb{L}$ of existential rules, the data complexity of consistent query answering under $\mathbb{L}[\bot]$ is coNP-hard

Data Complexity

class of $\exists$-rules complexity class

data complexity of classical query answering under $\mathbb{L}$ is $\mathbb{C}$-complete

$$\Downarrow$$

data complexity of consistent query answering under $\mathbb{L}[\perp]$ is:

coNP-complete if $\mathbb{C} \subseteq$ PTIME

From Classical to Consistent Query Answering

(ba-/fp)combined complexity:

in NP	$\rightarrow$	$\Pi_{P,2}$-complete
$\mathbb{C}$-complete, $\mathbb{C} \supseteq$ PSPACE & $\mathbb{C}$ is deterministic	$\rightarrow$	$\mathbb{C}$-complete

data complexity:

in $\mathbb{C} \subseteq$ PTIME	$\rightarrow$	coNP-complete

an (almost) complete picture for the main classes of existential rules + negative constraints

Existential Rules

$$\forall \mathbf{X}\ (\varphi(\mathbf{X}) \rightarrow \exists \mathbf{Y}\ (\psi(\mathbf{X},\mathbf{Y})))$$

conjunctions of atoms

- Classical query answering under existential rules is undecidable
  see, e.g., [Beeri & Vardi, ICALP 1981]

- Expressive decidable fragments - field of intense research

  (e.g., Montpellier, Dresden, Calabria, Oxford, Vienna, …)

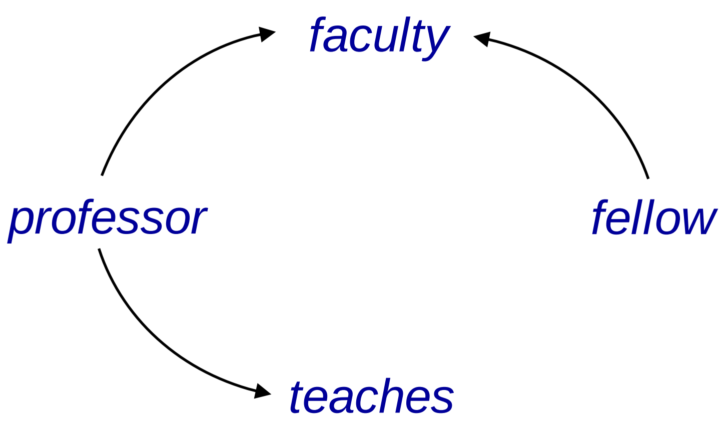
- Main decidability paradigms: acyclicity, guardedness & stickiness

Acyclic Existential Rules

- The predicate graph is acyclic

$$\forall X\,(professor(X) \;\rightarrow\; \exists Y\,(faculty(X) \wedge teaches(X,Y)))$$

$$\forall X\,(fellow(X) \;\rightarrow\; faculty(X))$$



(Frontier-)Guarded Existential Rules

- Frontier-guardedness: There exists a body-atom that contains the frontier

$$\forall X \forall Y \forall Z\, (\boldsymbol{supervisorOf(X,Y)} \wedge supervisorOf(Y,Z) \rightarrow manager(X))$$

- Guardedness: There exists a body-atom that contains all the $\forall$-variables

$$\forall X \forall Y\ (\boldsymbol{supervisorOf(X,Y)} \wedge emp(Y) \rightarrow emp\,(X))$$

- Linearity: There exists only one atom in the body

$$\forall X\ (\boldsymbol{employee(X)}\ \rightarrow\ \exists Y\ (supervisorOf(Y,X) \wedge employee(Y)))$$

Sticky Existential Rules

- Join-variables stick to the inferred atoms

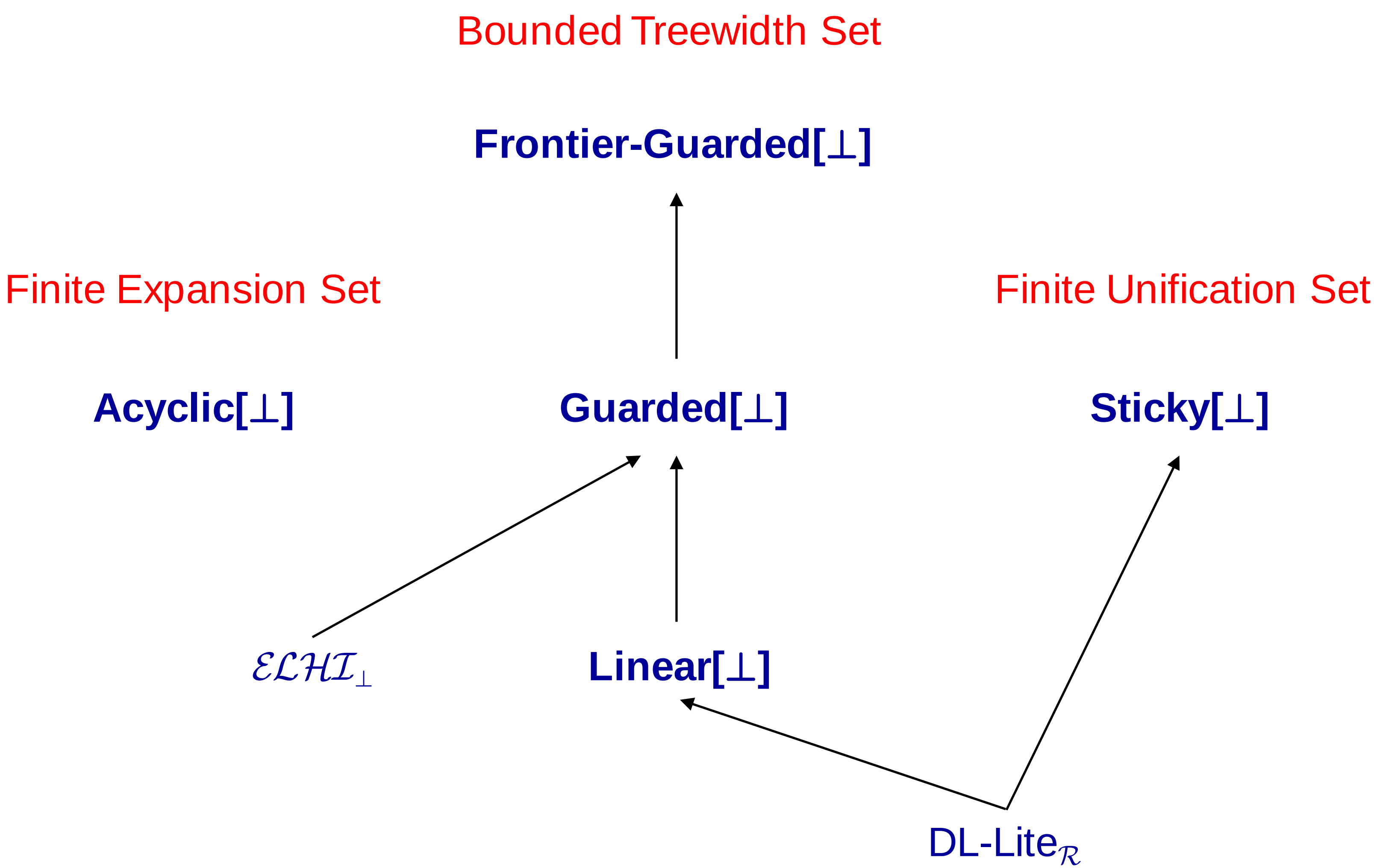
$$\forall X \forall Y \forall Z\, (q(X,Y) \wedge p(Y,Z) \rightarrow \exists W\, (t(X,Y,W)))$$

$$\forall X \forall Y \forall Z\, (t(X,Y,Z) \rightarrow \exists W\, (s(Y,W)))$$

$$\forall X \forall Y \forall Z\, (q(X,Y) \wedge p(Y,Z) \rightarrow \exists W\, (t(X,Y,W)))$$

$$\forall X \forall Y \forall Z\, (t(X,Y,Z) \rightarrow \exists W\, (s(X,W)))$$

Existential Rules + Negative Constraints



From Classical to Consistent Query Answering

(ba-/fp)combined complexity:

in NP $\rightarrow$ $\Pi_{p,2}$-complete

$\mathbb{C}$-complete, $\mathbb{C} \supseteq$ PSPACE & $\mathbb{C}$ is deterministic $\rightarrow$ $\mathbb{C}$-complete

data complexity:

in $\mathbb{C} \subseteq$ PTIME $\rightarrow$ coNP-complete

we simply need to exploit existing results on classical query answering

Classical Query Answering

	Combined	ba-combined	fp-combined	Data
Acyclic[⊥]	NEXPTIME	NEXPTIME	NP	in AC_0
Frontier-Guarded[⊥]	2EXPTIME	2EXPTIME	NP	PTIME
Guarded[⊥]	2EXPTIME	EXPTIME	NP	PTIME
Linear[⊥]	PSPACE	NP	NP	in AC_0
Sticky[⊥]	EXPTIME	NP	NP	in AC_0

Classical Query Answering

	Combined	ba-combined	fp-combined	Data
Acyclic[⊥]	NEXPTIME	NEXPTIME	NP	in AC_0
Frontier-Guarded[⊥]	2EXPTIME	2EXPTIME	NP	PTIME
Guarded[⊥]	2EXPTIME	EXPTIME	NP	PTIME
Linear[⊥]	PSPACE	NP	NP	in AC_0
Sticky[⊥]	EXPTIME	NP	NP	in AC_0

- Until recently, it was generally believed that it is EXPTIME
- The obvious algorithm does not work - models of double-exponential size

Classical Query Answering

	Combined	ba-combined	fp-combined	Data
Acyclic[⊥]	NEXPTIME	NEXPTIME	NP	in AC_0
Frontier-Guarded[⊥]	2EXPTIME	2EXPTIME	NP	PTIME
Guarded[⊥]	2EXPTIME	EXPTIME	NP	PTIME
Linear[⊥]	PSPACE	NP	NP	in AC_0
Sticky[⊥]	EXPTIME	NP	NP	in AC_0

- Upper bound: non-deterministically construct a proof of the query
- Lower bound: by reduction from a TILING problem

Classical Query Answering

	Combined	ba-combined	fp-combined	Data
Acyclic[⊥]	NEXPTIME	NEXPTIME	NP	in AC_0
Frontier-Guarded[⊥]	2EXPTIME	2EXPTIME	NP	PTIME
Guarded[⊥]	2EXPTIME	EXPTIME	NP	PTIME
Linear[⊥]	PSPACE	NP	NP	in AC_0
Sticky[⊥]	EXPTIME	NP	NP	in AC_0

(ba-/fp)combined complexity:

in NP $\rightarrow$ $\Pi_{P,2}$-complete

$\mathbb{C}$-complete, $\mathbb{C} \supseteq$ PSPACE & $\mathbb{C}$ is deterministic $\rightarrow$ $\mathbb{C}$-complete

data complexity:

in $\mathbb{C} \subseteq$ PTIME $\rightarrow$ coNP-complete

Consistent Query Answering

	Combined	ba-combined	fp-combined	Data
Acyclic[$\perp$]	?	?	$\Pi_{P,2}$	coNP
Frontier-Guarded[$\perp$]	2EXPTIME	2EXPTIME	$\Pi_{P,2}$	coNP
Guarded[$\perp$]	2EXPTIME	EXPTIME	$\Pi_{P,2}$	coNP
Linear[$\perp$]	PSPACE	$\Pi_{P,2}$	$\Pi_{P,2}$	coNP
Sticky[$\perp$]	EXPTIME	$\Pi_{P,2}$	$\Pi_{P,2}$	coNP

(ba-/fp)combined complexity:

in NP $\rightarrow$ $\Pi_{P,2}$-complete

$\mathbb{C}$-complete, $\mathbb{C} \supseteq$ PSPACE & $\mathbb{C}$ is deterministic $\rightarrow$ $\mathbb{C}$-complete

data complexity:

in $\mathbb{C} \subseteq$ PTIME $\rightarrow$ coNP-complete

Complexity of Acyclic[$\perp$]

- The guess and check algorithm gives a $\mathrm{coNP}^{\mathrm{NEXPTIME}}$ upper bound

- The class $\mathrm{NP}^{\mathrm{NEXPTIME}}$ lies at a higher level of the strong exponential hierarchy

- The SEH collapses to its Δ_2 level $\Rightarrow$ $\mathrm{NP}^{\mathrm{NEXPTIME}} = \mathrm{P}^{\mathrm{NE}}$
  [Hemachandra, J. Comput. Syst. Sci. 1989]

- P^{NE} is a deterministic class $\Rightarrow$ $\mathrm{coP}^{\mathrm{NE}} = \mathrm{P}^{\mathrm{NE}}$

Consistent Query Answering

	Combined	ba-combined	fp-combined	Data
Acyclic[⊥]	NEXP - P^{NE}	NEXP - P^{NE}	$\Pi_{P,2}$	coNP
Frontier-Guarded[⊥]	2EXPTIME	2EXPTIME	$\Pi_{P,2}$	coNP
Guarded[⊥]	2EXPTIME	EXPTIME	$\Pi_{P,2}$	coNP
Linear[⊥]	PSPACE	$\Pi_{P,2}$	$\Pi_{P,2}$	coNP
Sticky[⊥]	EXPTIME	$\Pi_{P,2}$	$\Pi_{P,2}$	coNP

$$P^{NE} \subseteq \text{coNEXPTIME}^{NP}$$

[Hemachandra, J. Comput. Syst. Sci. 1989]

Consistent Query Answering

	Combined	ba-combined	fp-combined	Data
Acyclic[⊥]	NEXP - P^{NE}	NEXP - P^{NE}	$\Pi_{P,2}$	coNP
Frontier-Guarded[⊥]	2EXPTIME	2EXPTIME	$\Pi_{P,2}$	coNP
Guarded[⊥]	2EXPTIME	EXPTIME	$\Pi_{P,2}$	coNP
Linear[⊥]	PSPACE	$\Pi_{P,2}$	$\Pi_{P,2}$	coNP
Sticky[⊥]	EXPTIME	$\Pi_{P,2}$	$\Pi_{P,2}$	coNP

Conjecture: Consistent query answering under Acyclic[⊥] is coNEXPTIME^{NP}-c

Data Intractable

	Combined	ba-combined	fp-combined	Data
Acyclic[⊥]	NEXP - P^{NE}	NEXP - P^{NE}	$\Pi_{P,2}$	coNP
Frontier-Guarded[⊥]	2EXPTIME	2EXPTIME	$\Pi_{P,2}$	coNP
Guarded[⊥]	2EXPTIME	EXPTIME	$\Pi_{P,2}$	coNP
Linear[⊥]	PSPACE	$\Pi_{P,2}$	$\Pi_{P,2}$	coNP
Sticky[⊥]	EXPTIME	$\Pi_{P,2}$	$\Pi_{P,2}$	coNP

but, what about tractability results w.r.t. the data complexity?

…consider approximations of the AR semantics

Intersection ABox Repair (IAR) Semantics

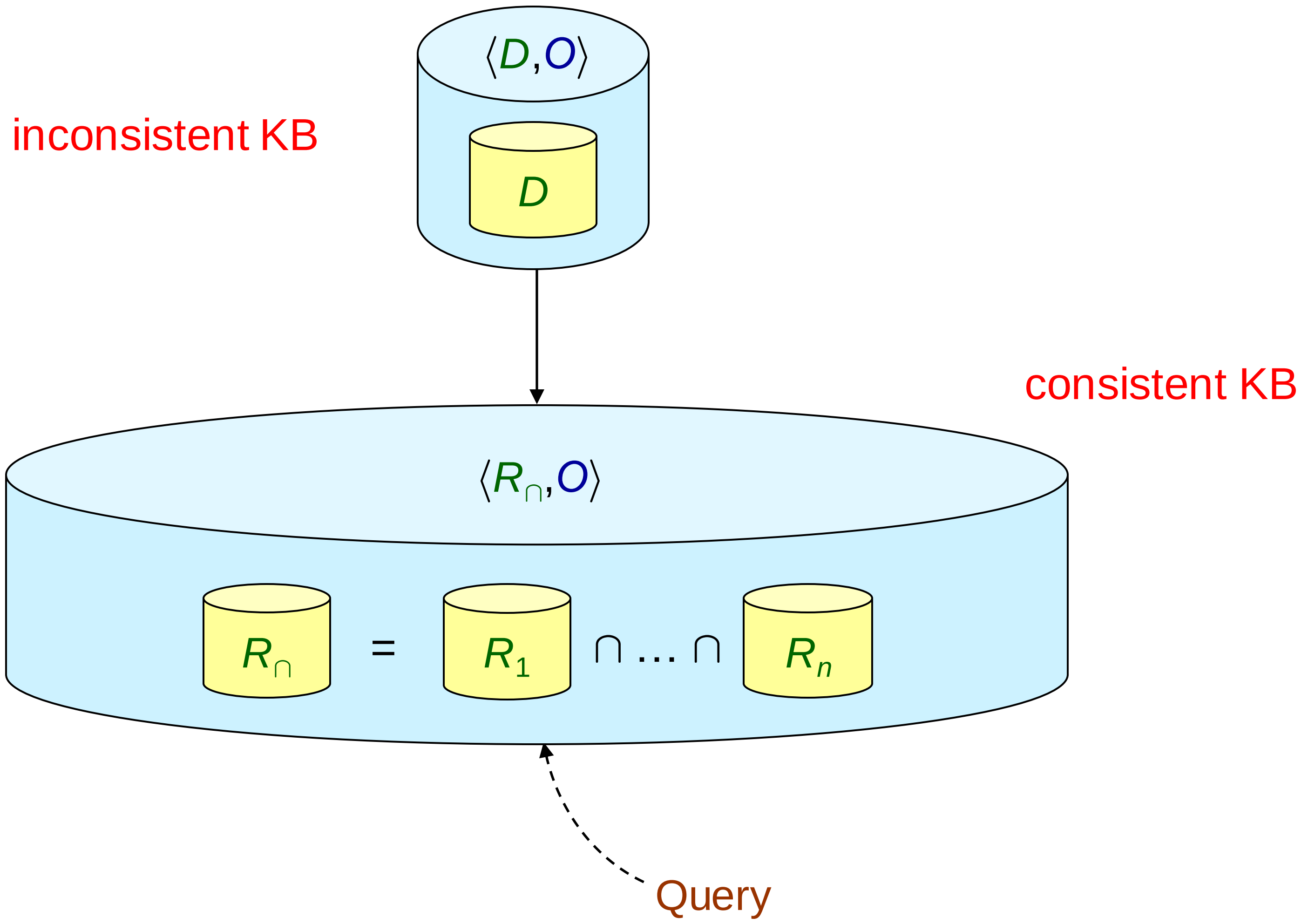
- One of the basic sound approximations of the AR semantics

- **IDEA:** The query must be entailed by the intersection of the database repairs

$\subseteq$-maximal consistent subsets of the database

[Lembo et al., RR 2010]

Intersection ABox Repair (IAR) Semantics



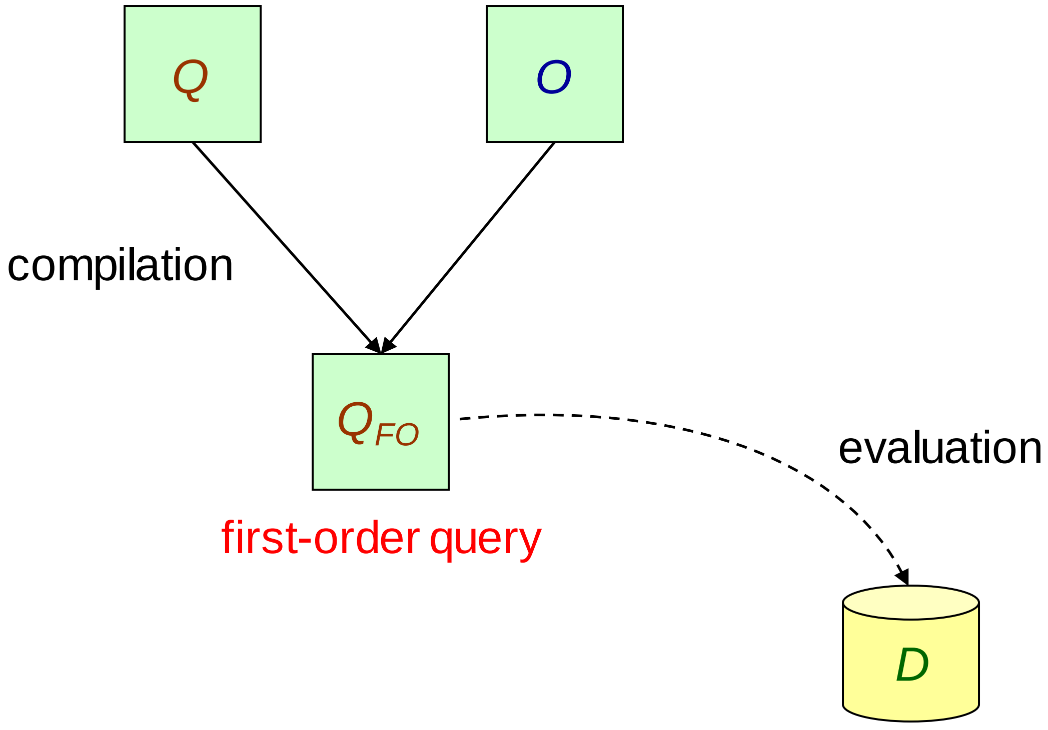
$$\langle D, O\rangle \models_{\mathrm{IAR}} \text{Query} \quad \Leftrightarrow \quad \langle R_\cap, O\rangle \models \text{Query}$$

Data Complexity under the IAR Semantics

Acyclic[⊥]	in AC_0
Frontier-Guarded[⊥]	coNP
Guarded[⊥]	coNP
Linear[⊥]	in AC_0
Sticky[⊥]	in AC_0

via first-order rewritability - a generic result can be established

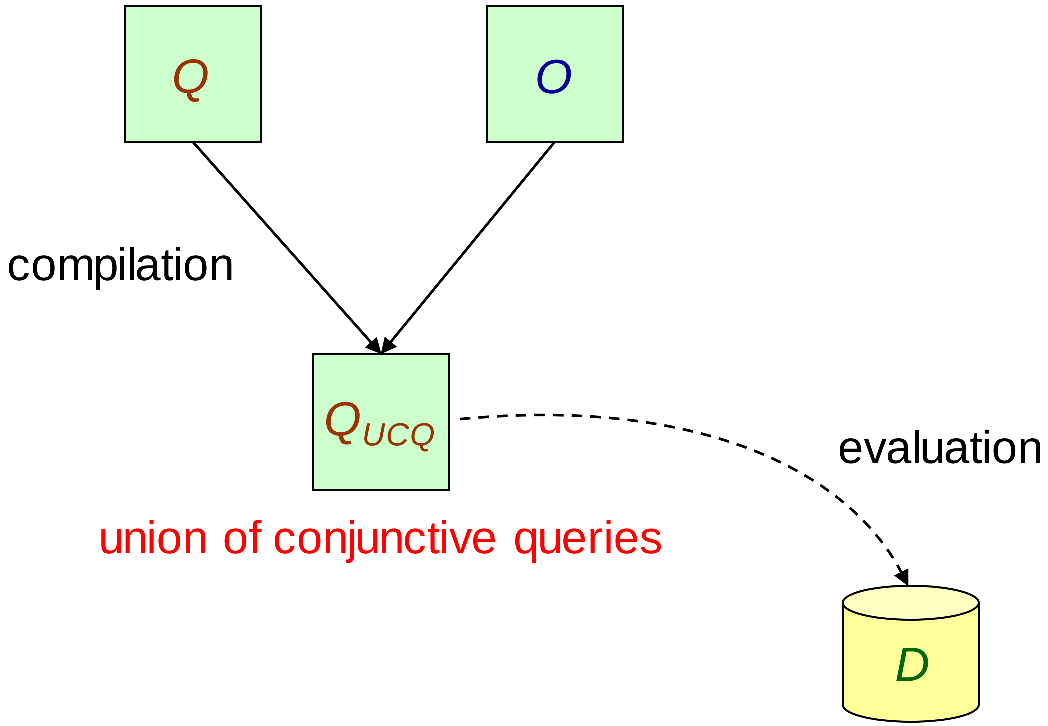
First-Order Rewritability (FO-Rewritability)



$$\forall D \;:\; \langle D, O \rangle \models Q \quad \Leftrightarrow \quad D \models Q_{FO}$$

$$\forall D \;:\; \langle D, O \rangle \models_{\mathrm{IAR}} Q \quad \Leftrightarrow \quad D \models Q_{FO}$$

UCQ-Rewritability



$$\forall D : \langle D, O \rangle \models Q \quad \Leftrightarrow \quad D \models Q_{UCQ}$$

$$\forall D : \langle D, O \rangle \models_{\mathrm{IAR}} Q \quad \Leftrightarrow \quad D \models Q_{UCQ}$$

From UCQ-Rewritability to FO-Rewritability

class of $\exists$-rules

classical query answering under $\mathbb{L}$ is UCQ-Rewritable

$\Downarrow$

consistent query answering under the IAR semantics for $\mathbb{L}[\perp]$ is FO-Rewritable

Data Complexity under the IAR Semantics

Acyclic[⊥]	in AC_0
Frontier-Guarded[⊥]	coNP
Guarded[⊥]	coNP
Linear[⊥]	in AC_0
Sticky[⊥]	in AC_0

via first-order rewritability - a generic result can be established

Key Message

We can transfer complexity results

from classical to consistent query answering

in a generic and uniform way

…with some unexpected exceptions - Acyclic[$\perp$]

Thank you!