From Classical to Consistent Query Answering under Existential Rules

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Ontology-based Query Answering (OBQA)

\[ \langle D, O \rangle \models \text{Query} \iff D \land O \models \text{Query} \]
A Simple Example

\[ D = \]

\[ \text{professor}(\text{John}) \]
\[ \text{fellow}(\text{John}) \]

\[ O = \]

\[ \forall X (\text{professor}(X) \rightarrow \exists Y (\text{faculty}(X) \land \text{teaches}(X,Y))) \]
\[ \forall X (\text{fellow}(X) \rightarrow \text{faculty}(X)) \]

\[ \forall M \models \langle D, O \rangle : M = \]

\[ \ldots \text{teaches}(\text{John},\#) \ldots \]

\[ \exists X (\text{teaches}(\text{John},X)) \quad \checkmark \]

\[ \{ \text{John} \rightarrow \text{John}, X \rightarrow \# \} \]
A Simple Example

\[ D = \]

- \texttt{professor(John)}
- \texttt{fellow(John)}

\[ O = \]

\[ \forall X (\text{professor}(X) \rightarrow \exists Y (\text{faculty}(X) \land \text{teaches}(X, Y))) \]
\[ \forall X (\text{fellow}(X) \rightarrow \text{faculty}(X)) \]
\[ \forall X (\text{professor}(X) \land \text{fellow}(X) \rightarrow \perp) \]

no model $\Rightarrow$ every query is entailed
Handling Data Inconsistencies

- The data are likely to be inconsistent with the ontology

- Standard semantics fails: everything is inferred - not meaningful answers

- Two approaches to inconsistency-handling:
  - Resolve the inconsistencies - ideal, but not always possible
  - Live with the inconsistencies - inconsistency-tolerant semantics
ABox Repair (AR) Semantics

• Standard inconsistency-tolerant semantics

• IDEA: The query must be entailed by every database repair
  \( \subseteq \)-maximal consistent subsets of the database

[Leombo et al., RR 2010]
ABox Repair (AR) Semantics

\[ \langle D, O \rangle \models_{AR} \text{Query} \iff \forall R \in \{R_1, \ldots, R_n\}: \langle R, O \rangle \models \text{Query} \]
ABox Repair (AR) Semantics: Example

\[ D = \]

- professor(John)
- fellow(John)

\[ O = \]

\[ \forall X (\text{professor}(X) \rightarrow \exists Y (\text{faculty}(X) \land \text{teaches}(X, Y))) \]

\[ \forall X (\text{fellow}(X) \rightarrow \text{faculty}(X)) \]

\[ \forall X (\text{professor}(X) \land \text{fellow}(X) \rightarrow \bot) \]

\[ \langle D, O \rangle \models_{AR} \text{faculty}(John) \]

\[ R_1 = \]

- professor(John)

\[ \langle R_1, O \rangle \models \text{faculty}(John) \]

\[ R_2 = \]

- fellow(John)

\[ \langle R_2, O \rangle \models \text{faculty}(John) \]
ABox Repair (AR) Semantics: Example

\[ D = \]

\[ \begin{align*} &\text{professor}(John) \\
&\text{fellow}(John) \end{align*} \]

\[ O = \]

\[ \begin{align*} &\forall X (\text{professor}(X) \rightarrow \exists Y (\text{faculty}(X) \land \text{teaches}(X,Y))) \\
&\forall X (\text{fellow}(X) \rightarrow \text{faculty}(X)) \\
&\forall X (\text{professor}(X) \land \text{fellow}(X) \rightarrow \bot) \end{align*} \]

\[ \langle D, O \rangle \models_{AR} \exists X (\text{teaches}(John,X)) \quad \times \]

\[ \langle R_1, O \rangle \models \exists X (\text{teaches}(John,X)) \quad \checkmark \]

\[ \langle R_2, O \rangle \models \exists X (\text{teaches}(John,X)) \quad \times \]
AR Semantics

- Lots of recent work and complexity results for description logics
  [Lembo et al., RR 2010 / Rosati, IJCAI 2011 / Bienvenu, AAAI 2012 / Bienvenu & Rosati, IJCAI 2013]

- This talk is about existential rules + negative constraints
  [Lukasiewicz, Martinez & Simari, ODBASE 2013 / Lukasiewicz, Martinez, P. & Simari, AAAI 2015]

\[
\forall X (\varphi(X) \rightarrow \exists Y (\psi(X,Y))) + \forall X (\varphi(X) \rightarrow \bot)
\]
Our Goal

Perform an in-depth complexity analysis of consistent query answering under the main classes of existential rules + negative constraints

- Combined
- Bounded-arity combined
- Fixed-program combined
- Data

generic complexity results - from classical to consistent query answering
Combined Complexity

\[ M \text{ complexity of classical query answering under } L \text{ is } C\text{-complete} \]

\[ \downarrow \]

\[ M \text{ complexity of consistent query answering under } L[\perp] \text{ is:} \]

\[ \Pi_{P,2}\text{-complete} \quad \text{if} \quad C = \text{NP} \]

\[ C\text{-complete} \quad \text{if} \quad C \supseteq \text{PSPACE} \text{ and } C \text{ is deterministic} \]
Combined Complexity: Upper Bounds

Guess and check algorithm (for the complement of the problem)

Input: $D, O \in \mathbb{L}[\bot], Q$

1. Guess $R \subseteq D$ - a possible repair

2. Verify that $R$ is a repair, i.e., $\langle R, O \rangle$ is consistent and $R$ is $\subseteq$-maximal

3. Verify that $\langle R, O \rangle$ does not entail $Q$

no harder than classical query answering under $\mathbb{L}$

$\Rightarrow$ our problem is in $\text{coNP}^C \Rightarrow$ in

\[
\begin{cases}
\text{coNP}^{NP} = \text{co}\Sigma_{P,2} = \Pi_{P,2} & \text{if } C = \text{NP} \\
\text{coNP}^C = \text{co}C = C & \text{if } C \supseteq \text{PSPACE} \text{ and } C \text{ is deterministic}
\end{cases}
\]
Combined Complexity

\[ \text{combined or ba-combined or fp-combined} \quad \text{class of } \exists\text{-rules} \quad \text{complexity class} \]

\[ M \text{ complexity of classical query answering under } L \text{ is } C\text{-complete} \]

\[ \downarrow \]

\[ M \text{ complexity of consistent query answering under } L[L[\bot]] \text{ is:} \]

\[ \Pi_{P,2}\text{-complete} \quad \text{if} \quad C = \text{NP} \]

\[ C\text{-complete} \quad \text{if} \quad C \supseteq \text{PSPACE} \& C \text{ is deterministic} \]
A Strong $\Pi_{P,2}$-hardness Result

Consistent query answering under the single constraint

$$\forall X \forall Y \forall Z \forall W \ (p(X,Y,Z) \land p(W,X,Z) \rightarrow \bot)$$

while the database and the query use only binary and ternary predicates

(by reduction from satisfiability of 2QBF formulas)

\[\downarrow\]

For every class $\mathcal{L}$ of existential rules, the fp-combined complexity of consistent query answering under $\mathcal{L}[\bot]$ is $\Pi_{P,2}$-hard
Combined Complexity

combined or ba-combined or fp-combined class of $\exists$-rules complexity class

$\mathcal{M}$ complexity of classical query answering under $\mathbb{L}$ is $\mathcal{C}$-complete

$\mathcal{M}$ complexity of consistent query answering under $\mathbb{L}[\perp]$ is:

$\Pi_{P,2}$-complete if $\mathcal{C} = \text{NP}$

$\mathcal{C}$-complete if $\mathcal{C} \supseteq \text{PSPACE}$ & $\mathcal{C}$ is deterministic
Data Complexity

Data complexity of classical query answering under $\mathbb{L}$ is $\mathbb{C}$-complete

$\Downarrow$

Data complexity of consistent query answering under $\mathbb{L}[\bot]$ is:

$\text{coNP}$-complete if $\mathbb{C} \subseteq \text{PTIME}$
Data Complexity: Upper Bounds

Guess and check algorithm (for the complement of the problem)

Input: $D$, $O \in \mathbb{L}[\bot]$, $Q$

1. Guess $R \subseteq D$ - a possible repair
2. Verify that $R$ is a repair, i.e., $\langle R, O \rangle$ is consistent and $R$ is $\subseteq$-maximal
3. Verify that $\langle R, O \rangle$ does not entail $Q$

no harder than classical query answering under $\mathbb{L}$

$\Rightarrow$ our problem is in $\text{coNP}^C \Rightarrow$ in $\text{coNP}$ (since $\text{NP}^{\text{PTIME}} = \text{NP}$)
A Strong coNP-hardness Result

Consistent query answering under the single constraint

\[ \forall X (p(X) \land s(X) \rightarrow \bot) \]

while the query is fixed

(by reduction from 2+2UNSAT)

\[ \Downarrow \]

For every class \( \mathbb{L} \) of existential rules, the data complexity of consistent query answering under \( \mathbb{L}[\bot] \) is coNP-hard
Data Complexity

data complexity of classical query answering under $L$ is $C$-complete

$\downarrow$

data complexity of consistent query answering under $L[\perp]$ is:

$\text{coNP-complete}$ if $C \subseteq \text{PTIME}$
From Classical to Consistent Query Answering

(ba-/fp)combined complexity:

\[(\text{in NP}) \rightarrow \Pi_{P,2}\text{-complete}\]
\[
\mathbb{C}\text{-complete}, \mathbb{C} \supseteq \text{PSPACE} \ & \ \mathbb{C} \text{ is deterministic} \rightarrow \mathbb{C}\text{-complete}
\]

data complexity:

\[\text{in } \mathbb{C} \subseteq \text{PTIME} \rightarrow \text{coNP-complete}\]

an (almost) complete picture for the main classes of existential rules + negative constraints
Existential Rules

\[ \forall X (\varphi(X) \rightarrow \exists Y (\psi(X,Y))) \]

conjunctions of atoms

- Classical query answering under existential rules is undecidable
  see, e.g., [Beeri & Vardi, ICALP 1981]

- Expressive decidable fragments - field of intense research
  (e.g., Montpellier, Dresden, Calabria, Oxford, Vienna, …)

- Main decidability paradigms: acyclicity, guardedness & stickiness
Acyclic Existential Rules

- The predicate graph is acyclic

\[ \forall X (\text{professor}(X) \rightarrow \exists Y (\text{faculty}(X) \land \text{teaches}(X, Y))) \]

\[ \forall X (\text{fellow}(X) \rightarrow \text{faculty}(X)) \]
(Frontier-)Guarded Existential Rules

- **Frontier-guardedness:** There exists a body-atom that contains the frontier

  \[ \forall X \forall Y \forall Z (\text{supervisorOf}(X, Y) \land \text{supervisorOf}(Y, Z) \rightarrow \text{manager}(X)) \]

- **Guardedness:** There exists a body-atom that contains all the \( \forall \)-variables

  \[ \forall X \forall Y (\text{supervisorOf}(X, Y) \land \text{emp}(Y) \rightarrow \text{emp}(X)) \]

- **Linearity:** There exists only one atom in the body

  \[ \forall X (\text{employee}(X) \rightarrow \exists Y (\text{supervisorOf}(Y, X) \land \text{employee}(Y))) \]
Sticky Existential Rules

- Join-variables **stick** to the inferred atoms

\[ \forall X \forall Y \forall Z (q(X,Y) \land p(Y,Z) \rightarrow \exists W (t(X,Y,W))) \]

\[ \forall X \forall Y \forall Z (t(X,Y,Z) \rightarrow \exists W (s(Y,W))) \]

\[ \forall X \forall Y \forall Z (q(X,Y) \land p(Y,Z) \rightarrow \exists W (t(X,Y,W))) \]

\[ \forall X \forall Y \forall Z (t(X,Y,Z) \rightarrow \exists W (s(X,W))) \]
Existential Rules + Negative Constraints

Finite Expansion Set

Acyclic[⊥]

Linear[⊥]

ELHI⊥

Guarded[⊥]

Bounded Treewidth Set

Frontier-Guarded[⊥]

Finite Unification Set

Sticky[⊥]

DL-LiteR
(ba-/fp)combined complexity:

\begin{align*}
\text{in NP} & \rightarrow \Pi_{p,2}\text{-complete} \\
\mathbb{C}\text{-complete, } \mathbb{C} & \supseteq \text{PSPACE} \quad \& \quad \mathbb{C} \text{ is deterministic} & \rightarrow \mathbb{C}\text{-complete}
\end{align*}

data complexity:

\begin{align*}
\text{in } \mathbb{C} & \subseteq \text{PTIME} & \rightarrow \text{coNP-complete}
\end{align*}

we simply need to exploit existing results on classical query answering
Classical Query Answering

<table>
<thead>
<tr>
<th>Class</th>
<th>Combined</th>
<th>ba-combined</th>
<th>fp-combined</th>
<th>Data</th>
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</thead>
<tbody>
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<td>Acyclic[⊥]</td>
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- Until recently, it was generally believed that it is EXPTIME
- The obvious algorithm does not work - models of double-exponential size
### Classical Query Answering

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- **Upper bound**: non-deterministically construct a proof of the query
- **Lower bound**: by reduction from a TILING problem
### Classical Query Answering

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**(ba-/fp)combined complexity:**

\[
\text{in NP} \rightarrow \Pi_{P,2}\text{-complete} \quad \text{C-complete, C} \supseteq \text{PSPACE} & \quad \text{C is deterministic} \rightarrow \text{C-complete}
\]

**data complexity:**

\[
\text{in C} \subseteq \text{PTIME} \rightarrow \text{coNP-complete}
\]
### Consistent Query Answering

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**Data complexity:**

- (ba-/fp)combined complexity:
  - in NP → Π<sub>P,2</sub>-complete
  - C-complete, C ⊃ PSPACE & C is deterministic → C-complete

- **Data complexity:**
  - in C ⊆ PTIME → coNP-complete
Complexity of Acyclic[\bot]

- The guess and check algorithm gives a $\text{coNP}^{\text{NEXPTIME}}$ upper bound

- The class $\text{NP}^{\text{NEXPTIME}}$ lies at a higher level of the strong exponential hierarchy

- The SEH collapses to its $\Delta_2$ level $\Rightarrow$ $\text{NP}^{\text{NEXPTIME}} = \text{P}^{\text{NE}}$
  

- $\text{P}^{\text{NE}}$ is a deterministic class $\Rightarrow$ $\text{coP}^{\text{NE}} = \text{P}^{\text{NE}}$
## Consistent Query Answering

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$P^{NE} \subseteq \text{coNEXPTIME}^{NP}$

Consistent Query Answering

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**Conjecture**: Consistent query answering under Acyclic[⊥] is coNEXPTIME^{NP-c}. 


Data Intractable

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but, what about tractability results w.r.t. the data complexity?

…consider approximations of the AR semantics
Intersection ABox Repair (IAR) Semantics

• One of the basic sound approximations of the AR semantics

• **IDEA:** The query must be entailed by the intersection of the database repairs \(\subseteq\)-maximal consistent subsets of the database

[Leombo et al., RR 2010]
Intersection ABox Repair (IAR) Semantics

\[ \langle D, O \rangle \models_{\text{IAR}} \text{Query} \iff \langle R_n, O \rangle \models \text{Query} \]
Data Complexity under the IAR Semantics

<table>
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</tbody>
</table>

via first-order rewritability - a generic result can be established
First-Order Rewritability (FO-Rewritability)

∀D : ⟨D,O⟩ ⊨ Q ⇔ D ⊨ Q_{FO}

∀D : ⟨D,O⟩ ⊨_{IAR} Q ⇔ D ⊨ Q_{FO}
UCQ-Rewritability

\[ \forall D : \langle D, O \rangle \models Q \iff D \models Q_{UCQ} \]

\[ \forall D : \langle D, O \rangle \models_{\text{IAR}} Q \iff D \models Q_{UCQ} \]
From UCQ-Rewritability to FO-Rewritability

class of $\exists$-rules

classical query answering under $\mathbb{L}$ is UCQ-Rewritable

\[\downarrow\]

consistent query answering under the IAR semantics for $\mathbb{L}[\bot]$ is FO-Rewritable
Data Complexity under the IAR Semantics

<table>
<thead>
<tr>
<th>Term</th>
<th>Complexity</th>
</tr>
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via first-order rewritability - a generic result can be established
Key Message

We can transfer complexity results from classical to consistent query answering in a generic and uniform way

…with some unexpected exceptions - Acyclic[⊥]

Thank you!