Logic Programming Techniques for Reasoning with Probabilistic Ontologies

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Outline

Introduction

- Semantic Web
  - Aims at making information available in a form that is understandable by machines
  - Web Ontology Language (OWL)
    - Based on Description Logics
- Reasoners
  - Most DL reasoners use a tableau algorithm for doing inference
  - Most of them are implemented in a procedural language
    - Example: Pellet, RacerPro, FaCT++

Uncertainty Representation

- Semantic Web
  - Incompleteness or uncertainty are intrinsic of much information on the World Wide Web
  - Most common approaches: probability theory, Fuzzy Logic
- Logic Programming
  - Uncertain relationships among entities characterize many complex domains
  - Most common approache: probability theory $\rightarrow$ Distribution Semantics [Sato, 1995].
    - It underlies many languages (ICL,PRISM, ProbLog, LPADs),...
    - They define a probability distribution over normal logic programs, called worlds
    - The distribution is extended to a joint distribution over worlds and queries
    - The probability of a query is obtained from this distribution by summing out worlds

DISPONTE: DIstribution Semantics for Probabilistic ONTologiEs

- Idea: **annotate axioms of an ontology with a probability** and assume that the axioms are pairwise independent

$$0.6 :: \mathit{Cat} \sqsubseteq \mathit{Pet}$$

- A probabilistic ontology defines thus a distribution over normal theories (worlds) obtained by including an axiom in a world with a probability given by the annotation

DISPONTE

- Atomic choice: a pair (E_i, k), where E_i is the ith probabilistic axiom and $k \in \{0, 1\}$ indicates whether E_i is chosen to be included in a world ($K = 1$) or not ($K = 0$).
- Selection σ: set of one atomic choice for each probabilistic axiom.
- σ identifies a world w_σ
- $P(w_\sigma) = \prod_{(E_i,1)\in\sigma} p_i \prod_{(E_i,0)\in\sigma}(1 - p_i)$
- Probability of a query Q given a world w: $P(Q|w) = 1$ if $w \models Q$, 0 otherwise
- Probability of Q
  $P(Q) = \sum_w P(Q, w) = \sum_w P(Q|w)P(w) = \sum_{w:w\models Q} P(w)$

Inference and Query answering

- The probability of a query Q can be computed according to the distribution semantics by first finding the explanations for Q in the knowledge base
- Explanation: subset of axioms of the KB that is sufficient for entailing Q
- All the explanations for Q must be found, corresponding to all ways of proving Q
- Probability of $Q \rightarrow$ probability of the DNF formula

$$F(Q) = \bigvee_{e \in E_Q} \left(\bigwedge_{F_i \in e} X_i \right)$$

  where E_Q is the set of explanations and X_i as a Boolean random variable associated to axiom F_i
- We exploit Binary Decision Diagrams for efficiently computing the probability of a DNF formula

BUNDLE

Binary decision diagrams for Uncertain reasoNing on Description Logic thEories

- BUNDLE performs inference over DISPONTE knowledge bases
- It exploits an underlying ontology reasoner able to return all explanations for a query, such as **Pellet** [Sirin et al., 2007].
- Explanations for a query in the form of *a set of sets of axioms*
- BUNDLE uses a *tableau algorithm*
- Each tableau expansion rule updates a tracing function τ, which associates sets of axioms with nodes and edges of the tableau

Non-determinism

- Problem: some tableau expansion rules are non-deterministic
  - Reasoners implement a search strategy in a or-branching space
- We want to find all the possible explanations for a query
  - The algorithm has to explore all the non-deterministic choices done

Why Prolog?

- The reasoners implemented using procedural languages have to implement also a backtracking algorithm to find all the possible explanations
  - Example: Pellet uses an hitting set algorithm that repeatedly removes an axiom from the KB and then computes again a new explanation
- Reasoners written in Prolog can exploit Prolog's backtracking facilities for performing the search

TRILL - Tableau Reasoner for descrIption Logics in proLog

- TRILL implements the tableau algorithm using Prolog
- It resolves the axiom pinpointing problem in which we are interested in the set of explanations that entail a query
- Thea2 library for converting OWL DL ontologies to Prolog:
  - each OWL axiom is translated into a Prolog fact
- It applies all the possible expansion rules, first the non-deterministic ones then the deterministic ones
- It returns the set of the explanations

TRILL - Tableau Reasoner for descrIption Logics in proLog

- Deterministic rules are implemented by predicates that take as input a tableau and return a new single tableau
- Non-deterministic rules are implemented by predicates that take as input a tableau and return a list of tableaux from which one is non-deterministically chosen.

TRILL^P - Tableau Reasoner for descrIption Logics in proLog exploting Pinpointing formula

- TRILL^P resolves the axiom pinpointing problem by computing a *pinpointing formula* [Baader and Peñaloza, 2010a, Baader and Peñaloza, 2010b]
  1. We associate a Boolean variable to each axiom of the KB
  2. The pinpointing formula is a monotone Boolean formula on these variables that compactly encodes the set of all explanations

TRILL^P - Tableau Reasoner for descrIption Logics in proLog exploting

Pinpointing formula

- Deterministic and non-deterministic rules are implemented in the same way of TRILL's expansion rules
- They associate a pinpointing formula to the labels of the nodes instead of a set of explanations

Computing the probability

- The pinpointing formula is a Boolean formula which can be directly translated into a BDD
- We can compute the probability of the query from the BDD as in BUNDLE

TRILL^P - Tableau Reasoner for descrIption Logics in proLog

- Example

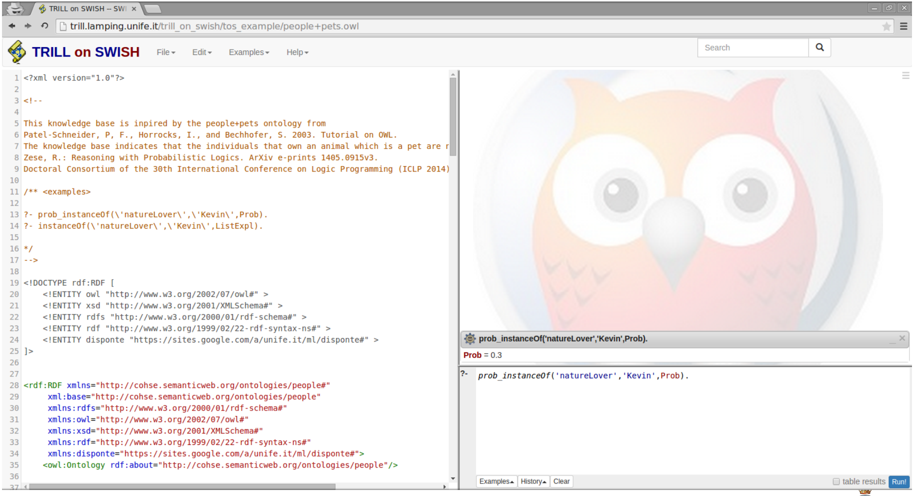
$$\begin{aligned} F_1 &= \mathit{fluffy} : \mathit{Cat} \\ F_2 &= \mathit{tom} : \mathit{Cat} \\ 0.6 :: F_3 &= \mathit{Cat} \sqsubseteq \mathit{Pet} \\ 0.5 :: F_4 &= \exists \mathit{hasAnimal}.\mathit{Pet} \sqsubseteq \mathit{NatureLover} \\ F_5 &= (\mathit{kevin}, \mathit{fluffy}) : \mathit{hasAnimal} \\ F_6 &= (\mathit{kevin}, \mathit{tom}) : \mathit{hasAnimal} \end{aligned}$$

- Let $Q = \mathit{kevin} : \mathit{NatureLover}$ be the query,
- the set of explanations is $\{\{F_5, F_1, F_3, F_4\}, \{F_6, F_2, F_3, F_4\}\}$,
- the pinpointing formula is $((F_5 \wedge F_1) \vee (F_6 \wedge F_2)) \wedge F_3 \wedge F_4$.
- the probability is $P = 0.3$

A Web Interface for TRILL: TRILL-on-SWISH

- SWISH [Lager and Wielemaker, 2014]
  - a recently proposed Web framework for logic programming
  - based on various features and packages of SWI-Prolog
  - allows the user to write Prolog programs and ask queries in the browser
- TRILL-on-SWISH allows users to write a KB in the RDF/XML format directly in the web page or load it from a URL, and specify queries that are answered by TRILL running on the server.
- Available at `http://trill.lamping.unife.it`.

TRILL-on-SWISH



Experiments

- Comparison between TRILL, TRILL^P and BUNDLE
- We consider four datasets:
  1. BRCA that models the risk factor of breast cancer;
  2. An extract of DBPedia;
  3. Biopax level 3 that models metabolic pathways;
  4. Vicodi that contains information on European history.

Table : Average time for computing the probability of queries in seconds.

DATASET	TRILL TIME (S)	TRILL^P TIME (S)	BUNDLE TIME (S)
BRCA	27.87	4.74	6.96
DBPedia	51.56	4.67	3.79
Biopax level 3	0.12	0.12	1.85
Vicodi	0.19	0.19	1.12

Conclusions

- We presented a semantics for modeling probabilistic DL KBs
- We presented three reasoners which can compute the probability of queries under the DISPONTE semantics
- We presented a web interface for TRILL, one of the reasoners presented in the paper
- The results we obtained show that:
  1. Prolog is a viable language for implementing DL reasoning algorithms
  2. TRILL's and TRILL^P's performances are comparable with those of a state-of-art reasoner

Thanks.

Questions?

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