Mapping Data to Ontologies With Exceptions Using Answer Set Programming

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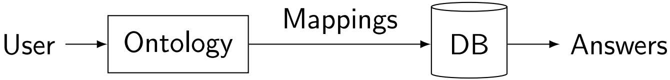
Ontology-Based Data Access



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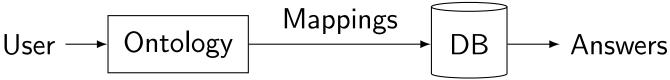

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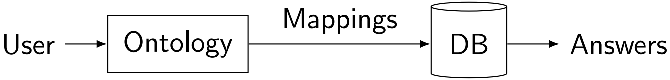

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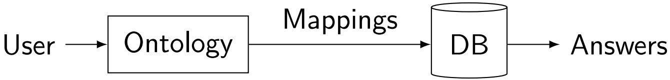
- In order to query a database, users can phrase queries in the ontology language
- These queries are then translated to database queries using the mappings
- Enables the use of a domain model that closely resembles end-users' understanding of a domain as opposed to complex and convoluted database schemas



- Formally, for $\varphi \rightsquigarrow \psi$:
  Given a database instance $\mathcal{D}$ and a set of mappings $\mathcal{M}$, $\mathcal{I}$ is a model of $(\mathcal{D}, \mathcal{M})$ if $\mathcal{I} \models \psi(\mathbf{t})$ for every query answer $\mathbf{t}$ of φ over $\mathcal{D}$.
- Mapping rewriting of an ontology query ψ w.r.t. $\mathcal{M}$: $\bigvee \varphi_i$ for every i with $\varphi_i \rightsquigarrow \psi \in \mathcal{M}$.

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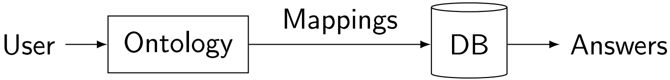

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Example:

$$\texttt{JOBS\_DB}(x, \text{``}\mathit{Accountant}\text{''}) \rightsquigarrow \mathit{Empl}(x)$$
$$\texttt{JOBS\_DB}(x, \text{``}\mathit{IT}\text{''}) \rightsquigarrow \mathit{Empl}(x)$$

then $\mathit{Empl}(x)$ would be rewritten to $\texttt{JOBS\_DB}(x, \text{``}\mathit{IT}\text{''}) \vee \texttt{JOBS\_DB}(x, \text{``}\mathit{Accountant}\text{''})$.

Limitations - OBDA

- databases typically use *closed-world (CW) reasoning*: if data cannot be explicitly found in the database, it is assumed to be false.

Person	Name
	Alice
	Bob
	Carla

$\rightarrow$ John is not a person, i.e., $\neg Person(John)$ is true.

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- ontologies employ *open-world (OW) reasoning*, where, in the above example, $\neg Person(John)$ could be either true or false
- mapping assertions $\varphi \rightsquigarrow \psi$ are interpreted as first-order implications, and thus inherently open-world!

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  $\rightarrow$ nonmonotonicity

Nonmonotonic Extensions

- adding nonmonotonicity to OBDA and description logic ontologies is an ongoing research topic, e.g., DL-programs [EIL+08], hybrid-MKNF knowledge bases [DNR02, MR10], closed predicates [LSW13].
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- However, the focus is on adding these capabilities to the ontologies
- propose extending mappings instead, as they are the tool used to connect closed-world and open-world

$\rightarrow$ *mapping programs*, an extension of $\exists$-ASP [GGLS15]

∃-ASP

An extension of classical ASP that supports

- existential quantification in the heads and negative bodies of rules,
- conjunctive queries in the heads and negative bodies of rules

An ∃-rule is of the form

$$H_1, \ldots, H_n \leftarrow B_1, \ldots, B_m, \\ \textbf{not}\,(C_1^1, \ldots, C_{u_1}^1), \ldots, \textbf{not}\,(C_1^s, \ldots, C_{u_s}^s).$$

where the H_i, B_j, C_k^l are atoms.

- all variables not occuring in the positive body are interpreted existentially.

∃-ASP

- due to the presence of existentials, only variables that are not existentials in negative bodies are grounded (*partial grounding*)
- the existential variables in the rule heads are *Skolemized*
- then the reduct and ∃-answer sets are defined anaologously to their classical ASP counterparts

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Theorem ([GGLS15])

For a given ∃-ASP program there exists an equivalent (w.r.t. answer sets) classical ASP program.

$\rightarrow$ reasoning in ∃-ASP can be reduced to reasoning in ASP

Mapping Programs

Extends ∃-ASP to allow for ontology queries in rule bodies

- A mapping rule is of the form

$$m : H^{\mathcal{T}}(\mathbf{x}, \mathbf{z}) \leftarrow \mathbf{not}\, J_1^-(\mathbf{y}_1), \ldots, \mathbf{not}\, J_k^-(\mathbf{y}_k),$$
$$J_1^+(\mathbf{y}'_1), \ldots, J_l^+(\mathbf{y}'_l), Q^{\mathcal{S}}(\mathbf{x}).$$

  where $Q^{\mathcal{S}}$ is a first-order formula over the database and $H^{\mathcal{T}}$, J_i^-, J_j^+ are first-order formulas over the ontology. The variables in $\mathbf{z}$ are existential variables, and $\mathbf{y}_i, \mathbf{y}'_j \subseteq \mathbf{x}$.

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- Intuitively, this can be read as follows:

  $Q^{\mathcal{S}}$ is mapped to $H^{\mathcal{T}}$ if all J_j^+ are certain answers and all J_i^- are not certain answers w.r.t. the mapping and ontology.

  J^+ and J^- are called the *positive* and *negative justifications*, respectively.

Example:
Let $\mathcal{D}$ consist of just one table, `JOBS_DB(<NAME>,<JOB>)`, and $\Sigma_{\mathcal{T}} = \{Empl, hasSup, depHeadOf\}$ be the signature of $\mathcal{T}$. The mapping rule

$$m_1 : \exists Z.hasSup(X, Z) \leftarrow \textbf{not}\, \exists Y.depHeadOf(X, Y), Empl(X), \texttt{Jobs\_DB}(X, P).$$

describes the default rule "employees, of whom we do not know that they are the head of a department, have a supervisor."

Mapping Programs — Skolemization

For a mapping rule

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define the *Skolem mapping rule* $sk(m)$ by replacing each existential variable $\mathbf{z}$ in $H^{\mathcal{T}}(\mathbf{x}, \mathbf{z})$ with a Skolem function symbol $sk_{\mathbf{z}}(s)$, where s is an ordered sequence of the universal variables $\mathbf{x}$.

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Definition (Skolem program [GGLS15])

For a mapping program $\mathcal{M}$, the set $sk(\mathcal{M}) = \{sk(m) \mid m \in \mathcal{M}\}$ is called the *Skolem program* of $\mathcal{M}$.

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$$\begin{aligned} sk(m_1) : hasSup(X, sk_z(X)) \leftarrow & \textbf{not } \exists Y.depHeadOf(X, Y), \\ & Empl(X), \texttt{Jobs\_DB}(X, P). \end{aligned}$$

Mapping Programs — Partial grounding

Definition (Partial ground programs, analogous to [GGLS15])

The *partial grounding* $PG(m)$ of a mapping rule m is the set of all partial ground instances of m over constants in $\Sigma_{\mathcal{D}}$ for those variables that are not existential variables in the J_i^-. The *partial ground program* of a mapping program $\mathcal{M}$ is the set $PG(\mathcal{M}) = \bigcup_{m \in \mathcal{M}} PG(m)$.

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Assume the constants occuring in the database are $\{a, b\}$, then $PG(sk(m_1))$ consists of the four mapping rules

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Mapping Programs — $\mathcal{T}$-Reduct

Since mapping rules include ontology predicates, we must take the ontology $\mathcal{T}$ into account when constructing the reduct:

Definition ($\mathcal{T}$-reduct)

Given an ontology $\mathcal{T}$, the $\mathcal{T}$-reduct $PG(\mathcal{M})^{\mathcal{A}}$ of a partial ground mapping program $PG(\mathcal{M})$ w.r.t. an interpretation $\mathcal{A}$ is the program obtained from $PG(\mathcal{M})$ after applying the following:

1. Remove all mapping rules m where there exists some $i \leq k$ such that $\mathcal{T} \cup \mathcal{A} \models J_i^-$.
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$\rightarrow$ the $\mathcal{T}$-reduct is a positive mapping program

Mapping Programs — Semantics

- A *mapping interpretation* $\mathcal{A}$ is a consistent subset of $HB_{sk(\mathcal{M})}$ (Herbrand base over $sk(\mathcal{M})$)
- $\mathcal{A}$ *satisfies the body* of a positive Skolemized mapping rule

$$sk(m) : H^{\mathcal{T}}(\mathbf{x}, sk_{\mathbf{z}}(\mathbf{x})) \leftarrow J_1^+(\mathbf{y}_1'), \ldots, J_l^+(\mathbf{y}_l'), Q^{\mathcal{S}}(\mathbf{x}).$$

  if the following holds: for every query answer $\mathbf{t}$ of $Q^{\mathcal{S}}$ over $\mathcal{D}$, every interpretation I with $I \models \mathcal{T} \cup \mathcal{A}$ satisfies $J_j^+[\mathbf{t}]$ for all $j \leq l$.
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Definition ($\mathcal{T}$-Answer Set)

A mapping interpretation $\mathcal{A} \subseteq HB_{sk(\mathcal{M})}$ is a $\mathcal{T}$*-answer set* of $\mathcal{M}$ if it is a $\subseteq$-minimal model of the reduct $PG(sk(\mathcal{M}))^{\mathcal{A}}$.

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Example:

Let $\mathcal{T} = \{Boss \sqsubseteq \exists depHeadOf\}$ and $\mathcal{M}$ consist of

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Then for $\mathcal{A} = \{\texttt{Jobs\_DB}(a, b), Empl(a), Boss(a)\}$, the rules

$$hasSup(a, sk_z(a)) \leftarrow \textbf{not}\, \exists Y.depHeadOf(a, Y), Empl(a), \texttt{Jobs\_DB}(a, v).$$

for $v \in \{a, b\}$ are removed from $PG(sk(\mathcal{M}))^{\mathcal{A}}$, since $\mathcal{T} \cup \mathcal{A} \models \exists Y.depHeadOf(a, Y)$.

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Mapping Programs in OBDA

Putting mapping programs into an OBDA context:

Definition (Generalized OBDA)

A *generalized OBDA specification* is a tuple $(\mathcal{D}, \mathcal{M}, \mathcal{T})$ consisting of a database $\mathcal{D}$, a mapping program $\mathcal{M}$, and an ontology $\mathcal{T}$.

Definition (Generalized OBDA semantics)

A tuple $(\mathcal{I}, \mathcal{A})$ consisting of a first-order model $\mathcal{I}$ and a mapping interpretation $\mathcal{A}$ is a model of $(\mathcal{D}, \mathcal{M}, \mathcal{T})$ if it satisfies the following:

1. $\mathcal{I} \models \mathcal{T} \cup \mathcal{A}$,
2. $\mathcal{A}$ is a $\mathcal{T}$-answer set of $\mathcal{M}$.

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Noteworthy:

- Due to $\mathbf{y}, \mathbf{y}' \subseteq \mathbf{x}$, $Q^{\mathcal{S}}(\mathbf{x})$ acts as a guard. Mapping rules are only applicable to tuples of constants from the database, not existential witnesses generated by mapping heads!

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- the partial grounding is always finite (for finite databases)
- Can express ontology constraints on the database: Let φ be the query to retrieve all tuples that are not in `JOBS_DB`. Then

  $$\perp \leftarrow \mathit{Person}(X), \varphi(X).$$

  expresses "All instances of *Person* must be contained in the table `JOBS_DB`."

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Theorem

The problem of checking $\mathcal{M} \models A$ for a given mapping program $\mathcal{M}$ and a ground atom A is undecidable.

Complexity

- Let $(\mathcal{T}, \mathcal{L})$ consist of an ontology $\mathcal{T}$ and a set $\mathcal{L}$ of formulas such that $\mathcal{T}$-entailment of any $\varphi \in \mathcal{L}$ is decided by an oracle $\mathcal{O}_{(\mathcal{T},\mathcal{L})}$.

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- Restrict $H^{\mathcal{T}}, J^{-}, J^{+}$ to formulas from $\mathcal{L}$.
- Then for a partially ground Skolem program $\mathcal{M}$, a guess-and-check algorithm can be used for $\mathcal{T}$-answer set construction: guess $\mathcal{A}$, construct the $\mathcal{T}$-reduct $\mathcal{M}^{\mathcal{A}}$, check satisfiability and minimality of $\mathcal{A}$.

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- Restrict $H^{\mathcal{T}}, J^{-}, J^{+}$ to formulas from $\mathcal{L}$.
- Then for a partially ground Skolem program $\mathcal{M}$, a guess-and-check algorithm can be used for $\mathcal{T}$-answer set construction: guess $\mathcal{A}$, construct the $\mathcal{T}$-reduct $\mathcal{M}^{\mathcal{A}}$, check satisfiability and minimality of $\mathcal{A}$.
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- generalization of the classical ASP guess-and-check: for $\mathcal{T} = \emptyset$ and $\mathcal{L} = \{\text{ground atoms}\}$, mapping programs are precisely ASP programs. $\rightarrow$ at least NP-hard (data complexity)

Complexity

More generally:

Theorem

Let $(\mathcal{T}, \mathcal{L})$ be a pair consisting of an first-order ontology $\mathcal{T}$ and a set of formulas $\mathcal{L}$ over the language of $\mathcal{T}$ such that $\mathcal{T}$-entailment is $|\mathcal{O}_{(\mathcal{T},\mathcal{L})}|$-hard for an oracle $\mathcal{O}_{(\mathcal{T},\mathcal{L})}$. Then for a partially ground Skolemized mapping program $\mathcal{M}$ where $H^{\mathcal{T}}, J^{-}, J^{+}$ are from $\mathcal{L}$, $\mathcal{T}$-answer set existence is $NP^{\mathcal{O}_{(\mathcal{T},\mathcal{L})}}$-complete.

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Proof idea:
Universal reduction argument: An $\mathrm{NP}^{\mathcal{O}_{(\mathcal{T},\mathcal{L})}}$ Turing machine can be encoded as a mapping program in the same manner an NP TM can be encoded in ASP, but allowing for oracle calls in rule bodies.

UCQ-Rewritability — Reduction to ASP

- The $\mathcal{T}$*-rewriting* of a query φ is a query $\overline{\varphi}$ such that
$$(\mathcal{D}, \mathcal{M}, \mathcal{T}) \models \varphi \text{ iff } (\mathcal{D}, \mathcal{M}, \emptyset) \models \overline{\varphi}.$$
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- Let $\mathcal{M}$ contain only rules where J^+, J^- are UCQ-rewritable w.r.t. $\mathcal{T}$ and $H^\mathcal{T}$ are conjunctive queries.
- Define $\overline{\mathcal{M}}$ as the program obtained by replacing J^+, J^- with their $\mathcal{T}$-rewritings $\overline{J^+}$ and $\overline{J^-}$.
- $\overline{\mathcal{M}}$ is equivalent to a $\exists$-program! (standard logic programming transformations)

UCQ-Rewritability — Reduction to ASP

Theorem

For a generalized OBDA specification $(\mathcal{D}, \mathcal{M}, \mathcal{T})$, where J^+, J^- in $\mathcal{M}$ are UCQ-rewritable with respect to $\mathcal{T}$, there exists an $\exists$-ASP program $\mathcal{M}'$ such that for a query q over $\mathcal{T}$

$$(\mathcal{D}, \mathcal{M}, \mathcal{T}) \vDash q[t] \iff \mathcal{M}' \vDash \overline{q}[t],$$

i.e., query answering over $(\mathcal{D}, \mathcal{M}, \mathcal{T})$ reduces to cautious reasoning over $\mathcal{M}'$.

Proof idea: Straightforward calculation.

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Proof idea: Straightforward calculation.
[GGLS15] $\Rightarrow$ can be further reduced to ASP.

Summary and Future Work

Summary:

- mapping programs as new mapping framework based on an extension of $\exists$-ASP.
- supports default exception handling and ontology constraints
- Algorithm for general, decidable case
- Reasoning reduces to ASP if the mapping program is UCQ-rewritable.

Future Work:

- analyze mapping programs from a parameterized complexity perspective
- determine which fragments of mapping programs admit a query rewriting process (currently not possible)
- Analyze when mapping programs can be rewritten to a classical OBDA mapping
- proof-of-concept implementation of an OBDA system using mapping programs

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